Detailed analysis of the safety of design buckling curves for steel members

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ABSTRACT: The development, harmonization and introduction of the unified Structural Eurocodes has arrived its final stage, incorporating a lot of research on re-examining and calibrating the basic standard procedures, and also introducing new, more refined ones. These recent research works focus mainly on the deterministic mechanical behavior of the standard models utilizing a great number of high-level numerical analyses performed on sophisticated, realistic models. Such a way the deterministic background of the standard is well verified, however the probabilistic verification of the formulas still needs research in order to make the refined design processes reliable and economic. The paper deals with the latter problem; we propose a probabilistic calibration method for the verification of resistance models in the standards utilizing fully the advantages of high-level numerical analysis. The method was applied to the probabilistic evaluation of design buckling curves as the most important basis for the design resistance of steel members.

1 INTRODUCTION

One of the most important application areas of probabilistic structural analysis is the reliability calibration of standard design methods and moreover the development of the safety concept and background behind the formulas. In the engineering practice the structural design standards provide the most frequently applied tool which guarantees the required safety level for the designer. Accordingly there have been developed many approaches for the calibration of different parts of design standards. These special purpose calibration methods are usually quite varying as they try to adapt to the characteristic signs of the examined task. This paper focuses on the probabilistic calibration procedure of design resistance formulas for steel structures. The unified regulations for the member states of the European Union are collected in the EN 1993 Europen Standard (2005). This part of the Structural Eurocodes is a progressively developing code including a great number of new design formulas (Boissonade et al. 2002, Greiner 2001). At this point some notes should be made in order to understand clearly the current situation around the calibration of these new resistance formulas:

1. In the earlier versions of the Structural Eurocodes the resistance formulas were usually calibrated directly to experimental results (Janns et al. 1992), however currently the new formulas developed on the basis of appropriate high level numerical analysis (Boissonade et al. 2002, Greiner 2001) exploiting the power of computer calculations;
2. Only deterministic calibration of the new design mechanical models is executed, a complete probabilistic verification – which shows the appropriate reliability level – is usually missing;
3. There is only one recommended statistical calibration method for the resistance formulas (Janns et al. 1992) which is rather an evaluation method of experimental test results – but sometimes applied for numerical results as well –, this method obviously does not take advantage of the opportunities implied in numerical calculations.

According to these notes in this paper a simple method is developed for the probabilistic evaluation of standard resistance models based on the results of numerical calculations (Szalai & Papp 2008). The method utilizes the features a design resistance model implies: description of only one failure mode, simple construction of resistance functions, the random variables at the resistance side have usually sufficient measured data for statistical analysis. The method also attempts to unify the accuracy level of the three main components of such calculations: (1) the deterministic model, (2) the probabilistic model, and (3) the set of random variables and their reliable statistics.

As an application example the design buckling curves for column and lateral-torsional buckling are evaluated as the most important basis for the stability design resistance of steel members. These curves are typical examples for the mentioned problems, as they were originally calibrated to tests by the recommended statistical evaluation method, later some modifications were added however without any probabilistic verification. Although, the column buckling is a deeply verified design problem (Beer & Schultz 1970, Galambos 1995, Maquoi & Rondal 1978), the widening knowledge of uncertainty data and the progressive methods for analysis of structural behavior allow but also obligate the engineer to reanalyze the standardized methods from time to time by applying the most up-to-date information and techniques. On the other hand it should be emphasized that the lateral-torsional buckling problem – which is far more difficult than the column buckling – has no such strong experimental and theoretical background resulting in a great deviation in the different national code regulations (Greiner 2001).The example is not sufficiently comprehensive to be eligible for a complete probabilistic verification of the resistance model, but gives a transparent overview about the working of the proposed method and the inconsistencies in the design curves.

2 THE PROBABILISTIC CALIBRATION MODEL

The main role of the probabilistic model in the present study is the calculation of the probabilistic quantiles of design resistances – these quantile values (often called characteristic or design values) are the most important parameters in the standard design process which control the safety level. For the calculation of quantiles the probabilistic distribution of the resistance should be assumed and also assumption for the distribution of random variables should be made. In the case of resistance calculations the random variables have two sources: geometrical and material parameters (including imperfections from both types). These parameters usually have a probabilistic distribution close to the Normal–type maybe including some skewness but extreme type distributions never occur. In these circumstances the method of moments yields accurate probabilistic estimates beside a simple, clear calculation scheme (Zhao & Ono 2001). Further advantage of the moment method is that it does not require the accurate probabilistic distribution of the random variables – which is usually not available or the determination is ambiguous – only the probabilistic moments of them. For the deterministic resistance model a special form response surface is applied. It was taken into account that the usual standard deviation of the random variables of the resistance model is quite small (the coefficient of variation (COV) never exceeds 0.1 in the examined cases), and within the small range of possible parameter values the resistance function behaves nearly linearly. Although this linear response was verified in terms of individual variables by pilot calculations (Szalai 2005) in case of the joint change of two or more variables the nonlinearity cannot be rejected so second order mixed terms are incorporated into the resistance model. The final shape of the resistance function is defined using the resistance points calculated by enough number of numerical analysis runs. Considering these thoughts, denoting the vector of the random variables by **X**, the resistance function in the space of the random variables is formulated as follows:

$$R(\mathbf{X}) = R(\mathbf{X}_0) + \mathbf{a}^T(\mathbf{X}-\mathbf{X}_0) + (\mathbf{X}-\mathbf{X}_0)^T\mathbf{B}(\mathbf{X}-\mathbf{X}_0) \quad (1)$$

where $\mathbf{X}_0$ is an arbitrary point in this space, and the coefficients can be written as:

$$a_j = \left.\frac{\partial R(\mathbf{X})}{\partial X_j}\right|_{\mathbf{X}=\mathbf{X}_0} \quad (2)$$

$$B_{jk} = \frac{1}{2}\left.\frac{\partial^2 R(\mathbf{X})}{\partial X_j \partial X_k}\right|_{\mathbf{X}=\mathbf{X}_0} \quad (3)$$

with $B_{jj} = 0$ for all j because of the discussed linear response. Such a way the resistance is described by a hyper second order saddle surface in the space of random variables.

After equating the point $\mathbf{X}_0$ to the point defined by the mean values of the random variables, special dimensionless sensitivity measures can be constructed incorporating the relative statistical behavior:

- first order sensitivity:

$$\Phi_j = \left.\frac{\partial R(\mathbf{X})}{\partial X_j}\right|_{\mathbf{X}=\mathbf{X}_0} \frac{X_{0j}}{R(\mathbf{X}_0)}\delta_j = a_j \frac{\mu_j}{\mu_R}\delta_j \quad (4)$$

- correlation sensitivity:

$$\Phi_{jk}^{corr} = a_j \frac{\mu_j}{\mu_R}\delta_j a_k \frac{\mu_k}{\mu_R}\delta_k \rho_{jk} = \Phi_j \Phi_k \rho_{jk} \quad (5)$$

- interaction sensitivity:

$$\Phi_{jk}^{int} = \left.\frac{\partial^2 R(\mathbf{X})}{\partial X_j \partial X_k}\right|_{\mathbf{X}=\mathbf{X}_0} \frac{X_{0j}X_{0k}}{R(\mathbf{X}_0)}\delta_j \delta_k =$$

$$= 2B_{jk}\frac{\mu_j \mu_k}{\mu_R}\delta_j \delta_k \quad (6)$$

where $X_{0j} = \mu_j$ and δ_j are the mean value and the COV of the j-th random variable, μ_R is the mean value of the resistance ($R(\mathbf{X}_0)$ – the resistance function at the mean values of the variables) and ρ_{jk} is the correlation coefficient between the j-th and k-th variable. Using these sensitivity factors, and applying the method of moments for the probabilistic description of the resistances, the first three moments of the resistance – having generally no reliable data for the fourth moment of either random variable – can be written as follows (Szalai 2005):

- mean:

$$\mu_R = R(\mathbf{X}_0) \tag{7}$$

- coefficient of variation (COV)

$$\delta_R = \frac{\sigma_R}{\mu_R} = \sqrt{\sum_{j=1}^{m} \Phi_j^2 + \sum_{j=1}^{m} \sum_{\substack{k=1 \\ k \neq j}}^{m} \Phi_{jk}^{corr} + \sum_{j=1}^{m} \sum_{k=1}^{m} \Phi_{jk}^{int\,2}} \tag{8}$$

- skewness:

$$\alpha_R = \frac{1}{\delta_R^3} \left(\sum_{j=1}^{m} \Phi_j^3 \alpha_j + 6 \sum_{j=1}^{m} \sum_{k=1}^{m} \Phi_j \Phi_k \Phi_{jk}^{int} \right) \tag{9}$$

where m is the total number of random variables and σ_R is the standard deviation of the resistance. In case of the skewness the effect of correlation has been neglected, because it would have required higher order correlation statistics, which is usually not available. Once these moments are available the required probabilistic values (characteristic or design quantiles) of the resistances can be calculated.

3 APPLICATION EXAMPLE

3.1 *Problem definition*

An application example is presented in order to demonstrate the practical working of the presented method. The probabilistic evaluation of the design buckling curves was chosen being basis for the stability design of steel members in the Structural Eurocodes.The flexural buckling (FB) and the lateral-torsional buckling (LTB) resistance of structural members are investigated, using the basic model for the standard design method: simply supported column with uniform compression, and simply supported beam with forked and free to warp ends loaded by uniform bending moment. Two commonly used hot rolled I profiles are applied, a slender (IPE 240) and a stocky (HEA 200) cross-section. For both sections and both loading cases eight different slenderness are examined to cover the whole practical range of the buckling curves.

3.2 *The deterministic resistance model*

For the numerical calculations a unique FE model was applied and developed specifically for steel thin-walled beam-column structures. The model was originally published by Rajasekaran & Murray (1973). It is built on a 15 DOF beam-column element, which can consider the overall geometric second order behavior, the very significant warping effect of the thin-walled cross-section and a multi-linear elastic material low. The method was improved by Papp et al. (2001), who developed a new thin-walled cross-section model, dividing the section into its segments thus making it possible to follow the real spread of yielding even along the thickness of the elements of the profile. The material law is generalized to real elasto-plastic kinematic strain hardening model, thus the presence of strain reversal in portions of the section can be taken into account. The solution process was also refined by Szalai & Papp (2005a) with an improved incremental-iterative method making the calculation with varying initial random parameters fully automatic and more efficient. For the determination of the coefficients of the response surface in all of the cases the resistances are calculated at the mean values of the variables and again at changing one variable with its standard deviation between proper limits (mean value then eight runs for each variable at +/-1, 2, 3 and 4 standard deviations shown in Figure 1).

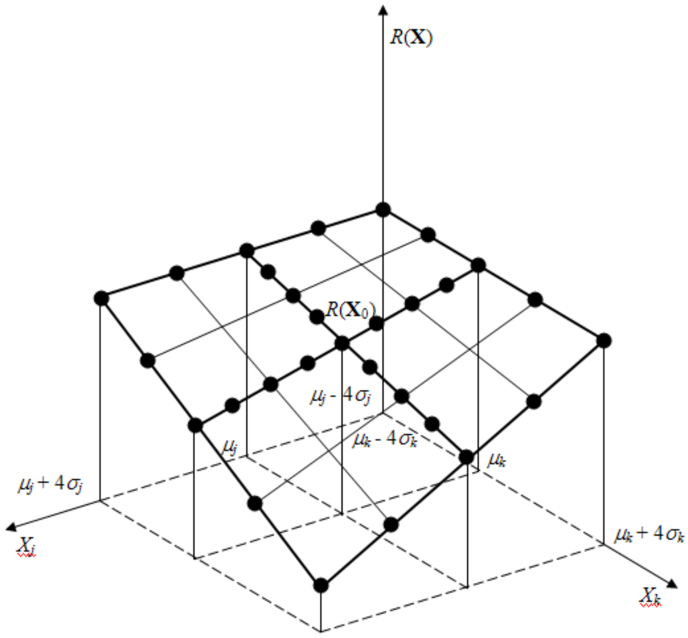

Figure 1. The calculation points determining the resistance function

3.3 *The random variables*

The random variables are selected considering two directives: they should deeply illustrate the probabilistic feature of the analyzed problems, and there should be a significant amount of statistical data for them to describe their necessary probabilistic parameters. The introduced deterministic model allows assuming a wide range of random variables so, after examining the stability problems and researching deeply in the literature to seek for possible and accessible statistical data, the parameters considered as random variables are established and grouped, they are summarized in Table 1. All the variables (except member out-of-straightness) are considered as constant throughout the member length. The geometric variables are the cross section dimensions, keeping the double symmetry of the shapes without the section imperfections and neglecting the rounding between the flanges and web. Since the probabilistic results are mainly described as a function of slenderness, the member length is taken as deterministic parameter. Based on preliminary sensitivity studies, only two variables are selected as random material parameters: the elastic modulus and the yield stress. Since in stability problems the yielding of the web is quite rare and insignificant before the loss of stability point, the statistical parameters of the yield stress are taken from the values obtained from the flanges (Melcher et al. 2004). One overall and three local geometric imperfections are taken into account. The initial out-of-straightness of the member is equal to a half-sine wave lateral deflection, and the variable is the midspan (maximum) value. The three local geometric imperfections are connected with the shape of the cross-section. The residual stress parameter (α) is the ratio of the residual stress at the tip of the flange and the yield stress assuming a suitable type distribution along the section (for hot-rolled profiles it is usually parabolic (Szalai & Papp 2005b).The statistical data for the random variables are carefully collected based on a widespread literature review which can be found in Szalai (2005) using partly own measurements, the values are shown in Table 2.

Table 1. The random variables

geometrical	material	imperfection
h-section height	E-elastic modulus	e_0-out-of-straightness
b-flange width	f_y-yield stress	k_0-section out-of-square
t_w-web thickness		f_0-web excentricity
t_f-flange thickness		c_0-web deformability
		α-residual stress

Table 2. The probabilistic parameters of random variables

variable	mean values		COVs	skewness
	IPE240	HEA200	(st.dev.)	
h[mm]	240	190	0.005	-0.50
b[mm]	120	200	0.01	-0.50
t_w*[mm]*	6.2	6.5	0.05	+0.4
t_f*[mm]*	9.8	10	0.05	+0.4
E[N/mm^2*]*	205000		0.04	-
f_y*[N/mm*^{2}*]*	280		0.07	+0.6
e_0*[mm]*	0.0008*L		0.15	-
k_0*[mm]*	0.0		(1.0)	-
f_0*[mm]*	0.0		(1.5)	-
c_0*[mm]*	0.0		(1.0)	-
α	0.2	0.3	0.25	-

Correlation between the geometric variables

	h	b	t_w	t_f
h	1	0.0534	0.0399	-0.0989
b	0.0534	1	-0.2142	-0.2681
t_w	0.0399	-0.2142	1	0.2451
t_f	-0.0989	-0.2681	0.2451	1

3.4 *The first order sensitivities*

For the analysis of the significance of the random variables the first order sensitivity factors are the most suitable measures. Firstly it is decided that which variables can be neglected without significant loss in the accuracy of the probabilistic parameters. It has been shown (Szalai 2005) that the web eccentricity (f_0) influences the COV of the resistances only if the load (normal force or bending moment) is applied on the web, which is not the case at practical structures; so the web eccentricity is neglected. When considering the height of the section (h), section out-of-square (k_0) and web deformability (c_0) the importance factor does not reach 1% for any of the cases, so these variables were also considered as deterministic variables with their mean values. The first order sensitivity factors of the rest of the random variables are shown in Figures 2-3 as a function of the lateral slenderness in the case of the two loading cases.

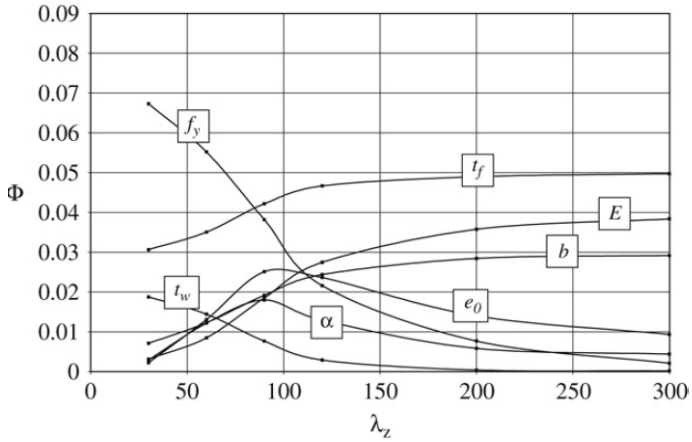

Figure 2. First order sensitivity factors for IPE 240 at flexural buckling

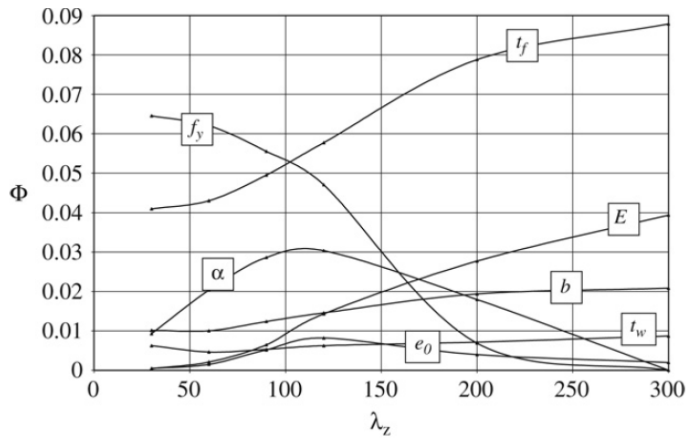

Figure 3. First order sensitivity factors for HEA 200 at lateral-torsional buckling

The diagrams clearly demonstrate the sensitivities and significances of the different variables as they are distributed along the slenderness. It can be readily seen, that the two most important variables are the thickness of the flange and the yield stress, while the least important is the thickness of the web. All of the variables have significant effect at FB in the medium slenderness range. It can also be noticed that the out-of-straightness and the elastic modulus are more important in the FB case. The IPE 240 section is more sensitive to the out-of-straightness, while residual stresses have greater influence in case of the HEA 200 profile. It is also important to note that in case of FB all the sensitivities approximate a constant value towards higher slenderness but in case of LTB the sensitivity factors of the flange thickness and elastic modulus show a progressively increasing trend.

3.5 *The physical interaction between variables*

There are two possible interactions between the variables: the physical and the probabilistic interaction. The latter one is known as correlation and represents some relationship between the probabilistic parameters of the individual variables. The physical interaction means that the effect of one variable on the resistance can change if the other variables have changed. In this study only the second order physical interaction is modeled in the resistance function by the mixed second order terms in Equation 1. Moreover only the second order interactions between the seven probabilistic variables were examined, the interactions between the four variables which were treated as deterministic in the previous section have been neglected. As a result of the analysis the interaction sensitivities according to Equation 6 were calculated, and it was concluded that neither of the factors exceeded the significant value, furthermore the sum of these factors – which reveals the significance of the total effect of interaction between all the variable pairs – showed also insignificant effect (Szalai & Papp 2009). According to these results it can be stated that the physical interaction between the random variables can be neglected, which means that the response surface can be modeled by a linear hyper plane without significant loss in the accuracy of probabilistic results.

3.6 *The correlation between variables*

The correlation can be easily considered in the described model; the main difficulty is that generally there are no reliable statistics for correlation coefficients. However for the geometrical variables of hot-rolled I sections (Melcher et al. 2004) some statistics were presented recently based on enough number of measurements including correlation coefficients, the values are collected in Table 1. Based on these values the following can be stated:

- the statistical relationship between the height of section and the other variables can be neglected;
- there is a significant negative correlation between the width of flange and the thickness of web and flange (the wider the flange the smaller the thickness);
- there is a significant positive correlation between the thickness of the web and flange.

These correlations were taken into the calculation, and moreover – although there are no statistical results – the possible negative and positive correlation between the yield stress and elastic modulus, and between the out-of-straightness and residual stress is analyzed considering a +/– 0.3 value for all cases.

The correlation sensitivities were calculated according to Equation 5. The effect of the correlation can be studied by calculating the COV of the resistances according to Equation 8 including the correlation term and excluding it (Figs 4-5).

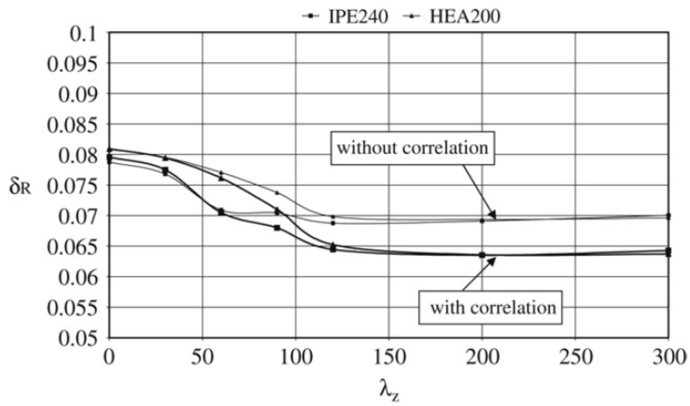

Figure 4. The coefficient of variation of the flexural buckling resistance

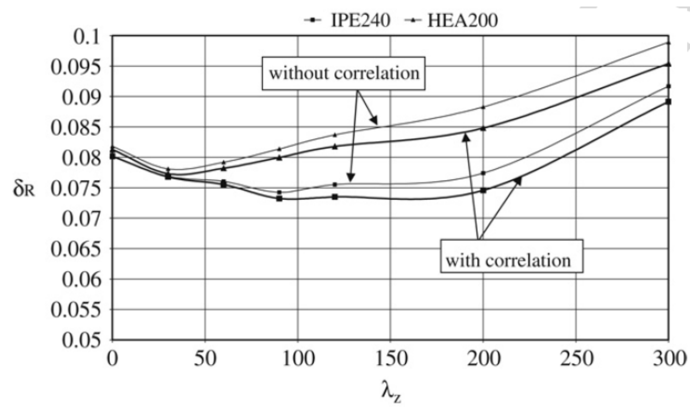

Figure 5. The coefficient of variation of the lateral-torsional buckling resistance

It can be seen that the LTB resistance has larger COV and a greater difference between the two types of the cross-sections. The most significant effect is the correlation between the width and thickness of the flange; it can reduce the COV of the FB resistance at higher slenderness even with 20%. The other two geometric correlations are much less important, although the correlation between the thicknesses can show higher values. The possible correlation between the material variables can be significant in the middle slenderness range, while the correlation between the imperfections is important only in case of the FB problem. Since there are no reliable data for the correlation between the material and imperfection variables we use only the geometric correlations in the further calculations. It is also worth noting that while at FB the COVs approximate a constant ~0.065 value towards higher slenderness, in case of LTB the COVs show significantly increasing character because of the increasing tendency of the sensitivities of the elastic modulus and the thickness of the flange (see Figs 2-3). As a summary it can be stated that in case of the FB resistance the COV varies between 0.06 and 0.08 with a decreasing tendency towards higher slenderness; and in case of the LTB resistance the COV varies between 0.07 and 0.1 with an increasing tendency towards higher slenderness.

3.7 *The effect of skewness*

According to Equation 9 the skewness of the resistance can arise from the skewness of the random variables or the nonlinearity of the resistance function (in that case the interaction sensitivity factors are nonzero, $\Phi_{jk}^{int} \neq 0$). Since in Section 3.5 it was concluded that the physical interaction between the variables can be neglected, only the variables with skew probabilistic distribution cause skewness in the resistance distribution. As it was presented in Table 2 there is reliable statistical data for the skewness only in case of the geometric variables and the yield stress. These values were taken into the formula of Equation 9 together with the corresponding first order sensitivity factors. The calculated skewness of the resistances is always positive, and varies between 0.17 and 0.43 in case of FB, and between 0.25 and 0.44 in case of LTB. In order to illustrate the effect of skewness on the safety of design formulas, the relative change of the design value of resistance (it corresponds to the 0.1% quantile in the EC3) using symmetric or skew distribution is presented. The quantiles of the symmetric distribution were calculated using the Normal distribution, while the Gamma distribution was used for considering the skewness (Szalai & Papp 2009). The Gamma distribution has been chosen since it approximates the Normal distribution as the skewness is tending to zero. The relative change in this quantile was calculated according to the following formula:

$$relative\ change = \frac{Q_{0.1\%}^{Gamma}}{Q_{0.1\%}^{Normal}} \qquad (10)$$

The relative change is shown in Figure 6, calculated at different skewness values in the practical range of COV. It can be seen that the calculated positive skewness always increases the quantile (the design value of the resistance), and the greater the COV and the skewness the greater the increase is. For both types of the examined problems the cross-section resistance (zero slenderness) has the greatest skewness (~0.44), and calculating with a δ_R = 0.8 COV (see Figs. 4-5) the increase in the design value is almost 7%. According to these points it can be stated that neglecting the skewness can cause significant change in the design value of the resistance although with increasing safety.

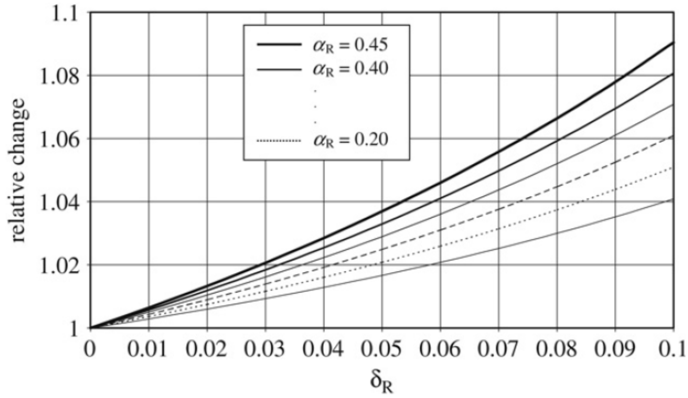

Figure 6. The relative change in the 0.1% quantile as a function of COV and skewness

3.8 *Safety of the buckling curves*

From the mean, COV and skewness using a three parameter Gamma distribution the probability of failure can be determined from the standard design resistance values and the calculated 0.1% quantiles – which are the required maximum risk of failure. The Figures 7-8 show these risk values in case of the two sections for the two buckling cases and assuming for the partial safety factor γ_{M1}=1.1 (earlier recommended value, dashed lines) and γ_{M1}=1.0 (new recommended value, solid lines).

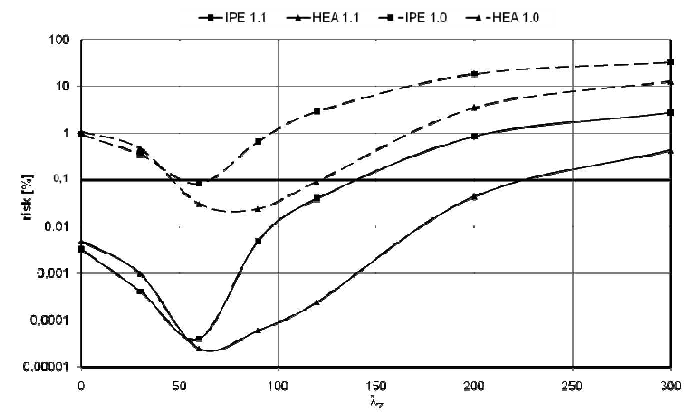

Figure 7. The risk of the failure of FB

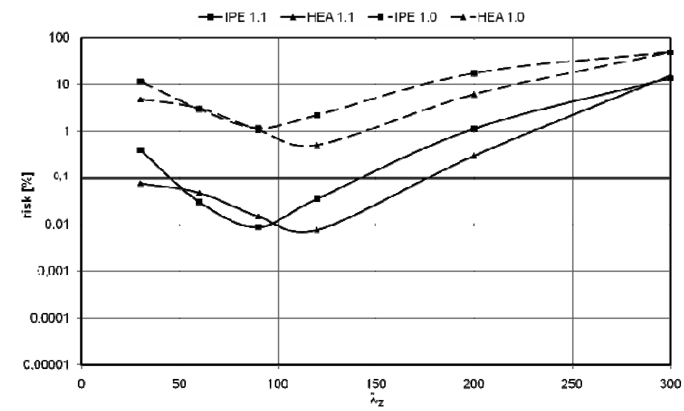

Figure 8. The risk of the failure of LTB

In case of FB the curve b corresponds to the IPE section and the curve c corresponds to the HEA section; in case of LTB the special curve is used for hot rolled sections (Section 6.3.2.3 in EN 1993 Europen Standard (2005)) with the recommended values. The first impression of the figures is that the use of the reduced 1.0 value for partial safety factor gives significantly unsafe results. These examples obviously cannot be considered as a base for making general conclusions; however a reassuring probabilistic verification of the reduced value has not been published yet. Another important conclusion is the common shape of the risk curves. These feature indicates that the buckling curves are not consistent (they do not provide the same probability against failure) along the slenderness. They start form a relatively high risk value, have the lowest – safest – values at the middle slenderness range, and have a progressively increasing portion towards higher slenderness. The main reason of this inconsistency in the safety level is that the shape of the buckling curves was calibrated to the deterministic behavior and in the Ayrton-Perry based resistance models the safety is incorporated only by the proper characteristic value of the yield stress, the increased imperfection factors and the partial safety factor (Janns et al. 1992). The geometrical properties and the elastic modulus appear with their nominal value – which generally takes a value around the mean – in the formula. However the dispersion of the geometrical variables and even the elastic modulus has significant effect on the resistance – as it was shown in Figures 2-3 –, and this effect varies differently against the slenderness than the yield stress and the imperfections. This is the reason for the progressive increase of risk in case of higher slenderness – where the effect of the yield stress and imperfections vanishes, but the influence of the flange thickness and elastic modulus rapidly increases. At middle slenderness the increased values of the imperfection parameters reduce the risk, while at low slenderness also the neglect of the effect of dispersion in the geometrical variables increases again the risk of failure. The presented application example indicates, that in line with the refined numerical analysis based deterministic calibration a deep probabilistic calibration is needed – examining the effects of all the important design parameters – for the correct determination of the design resistances in order to unify the safety level.

4 CONCLUSION

The objective of the paper is multiple. Firstly it attempts to complement a lack in the numerical analysis based standard calibration methods by proposing a special probabilistic calibration approach. The technique deeply illustrates the probabilistic characteristics of the numerical resistance models and applying the method of moments the probabilistic quantiles can be calculated and the safety of the standard models can be checked. A detailed application example is presented showing over the steps of the method examining the design buckling curves for the stability design of steel members. The example is obviously not widespread enough in point of the analyzed range of design cases to state general conclusions, but deep enough to point some features in the probabilistic behavior of numerical resistance models used for calibration and to reveal inherent discrepancies in the standard design models. Accordingly the goal of the example is mostly the demonstration of the practical working of the proposed method, but can also be used as a pilot examination or guideline for the more comprehensive numerical analysis based probabilistic evaluation of standard resistance formulas.

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